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Semisupermanifolds and Regularization of Categories, Modules, Algebras and Yang-Baxter Equation

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The concepts of regular n-cocycles, obstruction and the regularization procedure are developed as the category theory analogy of the semisupermanifolds with noninvertible transition functions. It is shown that the regularization of a category with nonivertible morphisms and obstruction form a 2-category. The generalization of some related structures to the regular case is considered.

In the category theory we study the obstructed cocycle conditions introduced earlier for noninvertible generalization of supermanifolds [1–3]. The concept of category regularization is considered. It is shown that the regularization of a category with noninvertible morphisms and obstruction forms a 2-category. Generalization of certain related structures as tensor operation, braidings, algebras and coalgebras etc... to our regular case is also given.

The standard patch definition of a supermanifold $\mathfrak{M}_0$ [4] is well-known [5]. Let $\bigcup\limits_{\alpha} \{U_\alpha, \varphi_\alpha\}$ is an atlas of a *supermanifold* $\mathfrak{M}_0$ [4], then transition functions satisfy the cocycle conditions $\Phi_{\alpha\beta}^{-1} = \Phi_{\beta\alpha}$on $U_\alpha \cap U_\beta$ and

$$\Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \Phi_{\gamma\alpha} = 1_{\alpha\alpha} \tag{1}$$

on triple overlaps $U_\alpha \cap U_\beta \cap U_\gamma$, where $1_{\alpha\alpha} \stackrel{\text{def}}{=} \text{id}\ (U_\alpha)$.

Definition 1. A *semisupermanifold* is a noninvertibly generalized superspace $\mathfrak{M}$ represented as a semi-atlas $\mathfrak{M} = \bigcup\limits_{\alpha} \{U_\alpha, \varphi_\alpha\}$ with invertible and noninvertible maps $\varphi_\alpha : U_\alpha \to V_\alpha \subset \mathbb{R}^{n|m}$ [3].

The semi-transition functions $\Phi_{\alpha\beta}$ of a semisupermanifold satisfy the following relations

$$\Phi_{\alpha\beta} \circ \Phi_{\beta\alpha} \circ \Phi_{\alpha\beta} = \Phi_{\alpha\beta} \tag{2}$$

on $U_\alpha \cap U_\beta$ overlaps and

$$\Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \Phi_{\gamma\alpha} \circ \Phi_{\alpha\beta} = \Phi_{\alpha\beta}, \tag{3}$$
$$\Phi_{\beta\gamma} \circ \Phi_{\gamma\alpha} \circ \Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} = \Phi_{\beta\gamma}, \tag{4}$$
$$\Phi_{\gamma\alpha} \circ \Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \Phi_{\gamma\alpha} = \Phi_{\gamma\alpha} \tag{5}$$

on triple overlaps $U_\alpha \cap U_\beta \cap U_\gamma$.

The relations (2)–(5) (we call them *tower relations*) satisfy identically in the standard invertible case [4]. The semisupermanifold defined above belongs to a class of so called obstructed semisupermanifolds [1,3] in the following sense. Let us rewrite relations (1) as the infinite series

$$n = 1: \quad \Phi_{\alpha\alpha} = 1_{\alpha\alpha}, \tag{6}$$
$$n = 2: \quad \Phi_{\alpha\beta} \circ \Phi_{\beta\alpha} = 1_{\alpha\alpha}, \tag{7}$$
$$n = 3: \quad \Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \Phi_{\gamma\alpha} = 1_{\alpha\alpha}, \tag{8}$$
$$n = 4: \quad \Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \Phi_{\gamma\delta} \circ \Phi_{\delta\alpha} = 1_{\alpha\alpha} \tag{9}$$

A semisupermanifold is called *obstructed*, if some of the cocycle conditions (6)–(9) are broken. It can happen that starting from some $n = n_m$ all higher cocycle conditions hold valid.

Definition 2. *Obstructedness degree* of a semisupermanifold is a maximal n_m for which the cocycle conditions (6)–(9) are broken. If all of them hold valid, then $n_m \stackrel{\text{def}}{=} 0$.

Ordinary manifolds [5] (with invertible transition functions) have vanishing obstructedness, and the obstructedness degree for them is equal to

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zero, i.e. $n_m = 0$. Therefore, using the obstructedness degree n_m, we have possibility to classify semisupermanifolds properly.

The above constructions have the general importance for *any* set of noninvertible mappings [6]. Therefore, by analogy with (2)–(5) it is natural to formulate the general

Definition 3. An noninvertible mapping $\Phi_{\alpha\beta}$ is *n-regular*, if it satisfies on overlaps $\overbrace{U_\alpha \cap U_\beta \cap \ldots \cap U_\rho}^{n}$ to the following conditions

$$\overbrace{\Phi_{\alpha\beta} \circ \Phi_{\beta\gamma} \circ \ldots \circ \Phi_{\rho\alpha} \circ \Phi_{\alpha\beta}}^{n+1} = \Phi_{\alpha\beta} + perm. \quad (10)$$

The formula (2) describes 3-regular mappings, the relations (3)–(5) correspond to 4-regular ones. Obviously that 3-regularity coincides with the ordinary regularity. The higher regularity conditions change dramatically the general diagram technique of morphisms, when we turn to noninvertible ones. Indeed, the commutativity of invertible morphism diagrams is based on the relations (6)–(9), i.e. on the fact that the tower identities are ordinary identities. When morphisms are noninvertible (a semisupermanifold has a nonvanishing obstructedness), we cannot "return to the same point", because in general $e_{\alpha\alpha}^{(n)} \neq 1_{\alpha\alpha}$, and we have to consider "nonclosed" diagrams due to the fact that the relation $e_{\alpha\alpha}^{(n)} \circ \Phi_{\alpha\beta} = \Phi_{\alpha\beta}$ is noncancellative now.

If we write higher n-regularity semicommutative diagrams, they can be considered in framework of generalized categories [7]. A category $\mathcal{C}$ contains a collection $\mathcal{C}_0$ of objects and a collection hom$(\mathcal{C})$ of arrows (morphisms) (see e.g. [8]). The collection hom$(\mathcal{C})$ is the union of mutually disjoint sets $\hom_{\mathcal{C}}(X,Y)$ of arrows $X \xrightarrow{f} Y$ from X to Y defined for every pair of objects $X, Y \in \mathcal{C}$. It may happen that for a pair $X, Y \in \mathcal{C}$ the set $\hom_{\mathcal{C}}(X,Y)$ is empty. The associative composition of morphisms is also defined. By an *equivalence* in $\mathcal{C}$ we mean a class of morphisms $\hom'(\mathcal{C}) = \bigcup_{X,Y\in(\mathcal{C}_0)} \hom'_{\mathcal{C}}(X,Y)$ where $\hom'_{\mathcal{C}}(X,Y)$ is a subset of $\hom_{\mathcal{C}}(X,Y)$. Two objects X, Y of the category $\mathcal{C}$ is equivalent if and only if there is an morphism $X \xrightarrow{s} Y$ in $\hom'_{\mathcal{C}}(X,Y)$ such that $s^{-1} \circ s = id_X$ and $s \circ s^{-1} = id_Y$. Our category can contains a class of *noninvertible* morphisms. A (strict) 2-category $\mathcal{C}$ consists of a collection $\mathcal{C}_0$ of objects as 0-cells and two collections of morphisms: $\mathcal{C}_1$ and $\mathcal{C}_2$ called 1-cells and 2-cells, respectively [9]. For every pair of objects $X, Y \in \mathcal{C}_0$ there is a category $\mathcal{C}(X,Y)$ whose objects are 1-cell $f : X \to Y$ in $\mathcal{C}_1$ and whose morphisms are 2-cells. For a pair of 1-cells $f, g \in \mathcal{C}_1$ there is a 2-cell $s : f \to g$ in $\mathcal{C}_2$. For every three objects $X, Y, Z \in \mathcal{C}_0$ there is a bifunctor $c := \{\mathcal{C}(X,Y) \times \mathcal{C}(Y,Z) \longrightarrow \mathcal{C}(X,Z)\}$ which is called a *composition* of 1-cells. There is an identity 1-cell $id_X \in \mathcal{C}(X,X)$ which acts trivially on $\mathcal{C}(X,Y)$ or $\mathcal{C}(Y,X)$. There is also 2-cell id_{id_X} which acts trivially on 2-cells.

Let $\mathcal{C}$ be a category with an equivalence. Then we can construct a 2-category $\mathbf{C}(\mathcal{C})$ whose 0-cells are equivalence classes of objects of $\mathcal{C}$, 1-cells are suitable classes of morphisms of $\mathcal{C}$, 2-cells are maps between these classes such that $\mathbf{C}(\mathcal{C})$ becomes a 2-category. Observe that 1-cells of $\mathbf{C}(\mathcal{C})$ can be represented by morphisms of the underlying category $\mathcal{C}$, but such representation is not unique. One can use 2-morphisms in order to change the representative morphism.

If the category $\mathcal{C}$ is equipped with certain additional structures, then they can be transforming into $\mathbf{C}(\mathcal{C})$. If for instance $\mathcal{C}$ is monoidal category with product $\otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$, then $\mathbf{C}(\mathcal{C})$ becomes the so-called semistrict monoidal 2-category. This means that the product $\otimes$ (under some natural conditions) is defined for all cells of the 2-category $\mathbf{C}(\mathcal{C})$. In the case of braided categories one can obtain the semistrict braided monoidal category [9]. Algebras, coalgebras, modules and comodules can be also included in this procedure. We apply such method to regularize categories with noninvertible morphisms and obstruction [6].

Let $\mathcal{C}$ be a category with invertible and noninvertible morphisms [6] and equivalence. The equivalence in $\mathcal{C}$ is here defined as the class of invertible morphisms in the category $\mathcal{C}$.

Definition 4. A sequence of morphisms

$$X_1 \xrightarrow{f_1} X_2 \xrightarrow{f_2} \ldots \xrightarrow{f_{n-1}} X_n \xrightarrow{f_n} X_1 \quad (11)$$

such that there is an (endo-)morphism $e_{X_1}^{(3)}$:

$X_1 \longrightarrow X_1$ defined uniquely by the following equation

$$e_{X_1}^{(n)} := f_n \circ \cdots \circ f_2 \circ f_1 \tag{12}$$

and subjects to the following relation

$$f_1 \circ f_n \circ \cdots \circ f_2 \circ f_1 = f_1$$

is said to be a *regular n-cocycle* on $\mathcal{C}$ and it is denoted by $f = (f_1, \dots f_n)$.

The (endo-)morphisms $e_{X_i}^{(n)} : X_i \longrightarrow X_i$ corresponding for $i = 2, \dots, n$ are defined by a suitable cyclic permutation of above sequence.

Definition 5. The morphism $e_X^{(n)}$ is said to be an obstruction of X. The mapping $e^{(n)} : X \in \mathcal{C}_0 \to e_X^{(n)} \in \hom(X,X)$ is called a regular n-cocycle obstruction structure on $\mathcal{C}$.

If $X_1 \xrightarrow{g_1} X_2' \xrightarrow{g_2} \cdots \xrightarrow{g_{n-1}} X_n' \xrightarrow{g_n} X_1$ is an another n-tuple of morphisms such that $e_{X_1}^{(n)} := g_n \circ \cdots \circ g_2 \circ g_1$, then we assume that X_i' is equivalent to X_i, for $i = 2, \dots, n$.

Definition 6. A map $s : f \Rightarrow g$ which sends the object X_i into equivalent object X_i' and morphism f_i into g_i is said to be obstruction n-cocycle equivalence.

We have the diagram

$$\begin{array}{ccccc} & & X_2 \xrightarrow{f_2} \cdots \xrightarrow{f_{n-1}} X_n & & \\ & \overset{f_1}{\nearrow} & & \overset{f_n}{\searrow} & \\ X_1 & & \Downarrow s & & X_1 \\ & \underset{g_1}{\searrow} & & \underset{g_n}{\nearrow} & \\ & & X_2' \xrightarrow{g_2} \cdots \xrightarrow{g_{n-1}} X_n' & & \end{array} \tag{13}$$

Lemma 7. *There is a one to one correspondence between equivalence classes of regular n-cocycles and regular n-cocycle obstruction structures.*

If $f = (f_1, \dots f_n)$ is a class of regular n-cocycles, then there is the corresponding regular n-cocycle obstruction structure $e^{(n)} : X \in \mathcal{C}_0 \to e_X^{(n)} \in \hom(X,X)$ such that the relation (12) holds true.

Let $e^{(n)} : X \in \mathcal{C}_0 \to e_X^{(n)} \in \hom(X,X)$ be a regular n-cocycle obstruction in $\mathcal{C}$.

Definition 8. A morphism $\alpha : X \longrightarrow Y$ of the category $\mathcal{C}$ such that

$$\alpha \circ e_X^{(n)} = e_Y^{(n)} \circ \alpha \tag{14}$$

is said to be a regular n-cocycle obstruction morphism from X to Y.

Definition 9. A collection of all equivalence classes of objects $\mathcal{C}_0$ with obstruction structures $e^{(n)} : X \in \mathcal{C}_0 \to e_X^{(n)} \in \hom(X,X)$ is denoted by $\Re eg_n(\mathcal{C})$ and called an obstruction n-cocycle regularization of $\mathcal{C}$. The class of all regular n-cocycle morphisms from X to Y is denote by $\Re eg_n(\mathcal{C})(X,Y)$.

Corollary 10. It follows from the Lemma 7 that the map $s : \alpha \longrightarrow \beta$ which sends an arbitrary regular n-cocycle morphisms $\alpha \in \Re eg_n(\mathcal{C})(X,X')$ into a regular n-cocycle morphisms $\beta \in \Re eg_n(\mathcal{C})(X,X')$ is a regular obstruction n-cocycle equivalence.

One can define 2-morphisms and an associative composition of 2-morphisms such that $\Re eg_n(\mathcal{C})(X,Y)$ becomes a category for every two objects $X, Y \in \mathcal{C}_0$. If $\alpha : X \longrightarrow Y$ and $\beta : Y \longrightarrow Z$ are two n-cocycle morphisms, then the composition $\beta \circ \alpha : X \to Z$ is also a n-cocycle morphism. In this way we obtain the composition as bifunctors

$$\begin{aligned} c^{\Re eg_n} \quad : \quad &= \{\Re eg_n(\mathcal{C})(X,Y) \times \Re eg_n(\mathcal{C})(Y,Z) \\ &\longrightarrow \Re eg_n(\mathcal{C})(X,Z)\} \end{aligned} \tag{15}$$

We summarize our considerations in the following lemma:

Lemma 11. *The class $\Re eg_n(\mathcal{C})$ forms a (strict) 2-category whose 0-cells are equivalence classes of objects of $\mathcal{C}$ with obstructions, whose 1-cells are regular n-cocycle obstruction morphisms, and whose 2-cells are regular obstruction n-cocycle 2-morphisms.*

Let $\mathcal{C} = \mathcal{C}(I, \otimes)$ be a monoidal category, where I is the unit object and $\otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$ is the monoidal product [10]. If the following relation

$$e_X^{(n)} \otimes e_Y^{(n)} = e_{X \otimes Y}^{(n)}. \tag{16}$$

holds true, then we have

Proposition 12. *The monoidal product of two regular n-cocycles $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ with obstruction $e^{(n)}_{X_1}$, and $e^{(n)}_{Y}$, respectively, is the regular n-cocycle $X_1 \otimes Y_1, \otimes \cdots \otimes X_n \otimes Y_n$ with the obstruction $e^{(n)}_{X\otimes Y}$.*

One can see that in this case $\Re eg_n(\mathcal{C})$ is the so-called semistrict monoidal category [9].

Let $\mathcal{C}$ and $\mathcal{D}$ be two monoidal categories and let $\Re eg_n(\mathcal{C}), \Re eg_n(\mathcal{D})$ be their regularization 2-categories. We can introduce the notion of regular 2-functors, pseudonatural transformations and modifications. All definitions do not changed, but the preservation of the identity can be replaced by the requirement of preservation of obstruction morphisms $e^{(n)}_X$ and the invertibility is replaced by regularity. If, for instance, there is a regular 2-functor $\mathcal{F} : \Re eg_n(\mathcal{C}) \longrightarrow \Re eg_n(\mathcal{D})$, then in addition to the standard definition [8] we have the following relation

$$\mathcal{F}(e^{(n)}_X) = e^{(n)}_{\mathcal{F}(X)}. \tag{17}$$

In the same manner we can "regularize" pseudo-natural transformations and modifications. Let $\Re eg_n(\mathcal{C})$ be a semistrict monoidal 2-category. A pseudo-natural transformations $B = \{B_{X,X'} : X \otimes X' \to X' \otimes X\}$ and two regular modifications $B_{X\otimes Y,Z}$, $B_{X,Y\otimes Z}$ such that

$$\begin{array}{ccc} X\otimes Y\otimes Z & \overset{B_{X\otimes Y,Z}}{\longrightarrow} & Y\otimes Z\otimes X \\ B_{X,Y}\otimes e^{(n)}_Z \searrow & & \nearrow e^{(n)}_Y \otimes B_{X,Z} \\ & Y\otimes X\otimes Z & \end{array} \tag{18}$$

and

$$\begin{array}{ccc} X\otimes Y\otimes Z & \overset{B_{X,Y\otimes Z}}{\longrightarrow} & Z\otimes X\otimes Y \\ e^{(n)}_X\otimes B_{Y,Z} \searrow & & \nearrow B_{X,Z}\otimes e^{(n)}_Y \\ & X\otimes Z\otimes Y & \end{array} \tag{19}$$

and

$$B_{X,X'} \circ e^{(n)}_{X\otimes X'} = e^{(n)}_{X'\otimes X} \circ B_{X,X'}, \tag{20}$$

are said to be a regular n-cocycle braiding. Obviously, these operations must satisfying all conditions of [9] with two indicated above changes.

Then the 2-category $\Re eg_n(\mathcal{C})$ is called a semistrict regular n-cocycle braided monoidal category.

We obtain here the following regular n-cocycle Yang–Baxter equation [6]

$$\begin{aligned} &\mathbf{B}^{(n)\,\mathrm{L}}_{Y,Z,X} \circ \mathbf{B}^{(n)\,\mathrm{R}}_{Y,X,Z} \circ \mathbf{B}^{(n)\,\mathrm{L}}_{X,Y,Z} \\ = \;&\mathbf{B}^{(n)\,\mathrm{R}}_{Z,X,Y} \circ \mathbf{B}^{(n)\,\mathrm{L}}_{X,Z,Y} \circ \mathbf{B}^{(n)\,\mathrm{R}}_{X,Y,Z}, \end{aligned} \tag{21}$$

where the notations $\mathbf{B}^{(n)\,\mathrm{L}}_{X,Y,Z} = B_{X,Y} \otimes e^{(n)}_Z$, $\mathbf{B}^{(n)\,\mathrm{R}}_{X,Y,Z} = e^{(n)}_X \otimes B_{Y,Z}$ have been used and the obstructors $e^{(n)}_X$ are exploited instead of the identity id_X. Solutions of the regular n-cocycle Yang–Baxter equation (21) can be found by application of the semigroup methods used in [11].

Let $\mathcal{C}$ be a monoidal category and $\Re eg_n(\mathcal{C})$ be its regularization . It is known [8] that an associative algebra in the category $\mathcal{C}$ is an object $\mathcal{A}$ of this category such that there is an associative multiplication $\mathrm{m} : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$ which is also a morphism of this category. If the multiplication is in addition a regular n-cocycle morphism, then the algebra $\mathcal{A}$ is said to be a *regular n-cocycle algebra*. This means that we have the relation

$$\mathrm{m} \circ (e^{(n)}_{\mathcal{A}} \otimes e^{(n)}_{\mathcal{A}}) = e^{(n)}_{\mathcal{A}} \circ \mathrm{m}. \tag{22}$$

Obviously such multiplication not need to be unique. Denote by $\Re eg_n(\mathcal{C})(\mathcal{A} \otimes \mathcal{A}, \mathcal{A})$ a class of all such multiplications. We can see that a regular n-cocycle 2-morphisms $s : \mathrm{m} \Rightarrow \mathrm{m}'$ which send the multiplication m into a new one m′ should be an algebra homomorphism. One can define regular n-cycle coalgebra or bialgebra in a similar way. A comultiplication $\Delta : \mathcal{A} \longrightarrow \mathcal{A} \otimes \mathcal{A}$ can be regularized according to the relation

$$\Delta \circ e^{(n)}_{\mathcal{A}} = (e^{(n)}_{\mathcal{A}} \otimes e^{(n)}_{\mathcal{A}}) \circ \Delta. \tag{23}$$

In this case we obtain a class $\Re eg_n(\mathcal{C})(\mathcal{A}, \mathcal{A}\otimes\mathcal{A})$ of comultiplications.

Let $\mathcal{A}$ be a regular n-cocycle algebra. If $\mathcal{A}$ is also regular coalgebra such that $\Delta(ab) = \Delta(a)\Delta(b)$, then it is said to be a *regular n-cocycle almost bialgebra.* If $\mathcal{A}$ is a regular n-cocycle algebra, then we denote by $\mathrm{hom}_{\mathrm{m}}(\mathcal{A}, \mathcal{A})$ the set of morphisms $s \in \mathrm{hom}_{\mathcal{C}}(\mathcal{A}, \mathcal{A})$ satisfying the condition

$$s \circ \mathrm{m} = \mathrm{m} \circ (s \otimes s). \tag{24}$$

Let $\mathcal{A}$ be a regular n-cocycle almost bialgebra. We define the *convolution product*

$$s \star t := \mathrm{m} \circ (s \otimes t) \circ \Delta, \tag{25}$$

where $s, t \in \mathrm{hom_m}(\mathcal{A}, \mathcal{A})$. If $\mathcal{A}$ is a regular n-cocycle almost bialgebra, then the convolution product is regular. A regular n-cocycle almost bialgebra $\mathcal{H}$ equipped with an element $S \in \mathrm{hom_m}(\mathcal{H}, \mathcal{H})$ such that

$$S \star id_{\mathcal{H}} \star S = S, \quad id_{\mathcal{H}} \star S \star id_{\mathcal{H}} = id_{\mathcal{H}}. \tag{26}$$

is said to be a *regular n-cocycle almost Hopf algebra $\mathcal{H}$*. This is a regular analogy of week Hopf algebras considered in [12] (see also [11]).

Let ${}_{\mathcal{A}}\mathcal{C}$ be a category of all left $\mathcal{A}$ -modules, where $\mathcal{A}$ is a bialgebra. For the regularization $\Re eg_n({}_{\mathcal{A}}\mathcal{C})$ of the $\mathcal{A}$–module action $\rho_M : \mathcal{A} \otimes M \longrightarrow M$ we use the following formula

$$\rho_M \circ (e_{\mathcal{A}}^{(n)} \otimes e_M^{(n)}) = e_M^{(n)} \circ \rho_M, \tag{27}$$

where $\rho_M : \mathcal{A} \otimes M \longrightarrow M$ is the left module action of $\mathcal{A}$ on M. The class of all such module actions is denoted by $\Re eg_n({}_{\mathcal{A}}\mathcal{C})(\mathcal{A} \otimes \mathcal{M}, \mathcal{M})$. The monoidal operation in this category is given as the following tensor product of $\mathcal{A}$-modules

$$\begin{aligned} \rho_{M \otimes N} \quad : \quad &= (id_M \otimes \tau \otimes id_N) \circ (\rho_M \otimes \rho_N) \\ &\circ (\Delta \otimes id_{M \otimes N}), \end{aligned} \tag{28}$$

where $\tau : \mathcal{A} \otimes M \to M \otimes \mathcal{A}$ is the twist, i. e. $\tau(a \otimes m) := m \otimes a$ for every $a \in \mathcal{A}$, $m \in M$.

Lemma 13. *For the tensor product of module actions we have the following formula*

$$\rho_{M \otimes N} \circ (e_A \otimes e_{M \otimes N}) = e_{M \otimes N} \circ \rho_{M \otimes N}. \tag{29}$$

Let $\mathcal{C}^{\mathcal{A}}$ be a category of right $\mathcal{A}$-comodules, where $\mathcal{A}$ is an algebra. The corresponding regularization can be given by the formulae

$$\begin{aligned} &\rho \circ e_{\mathcal{A}}^{(n)} = (e_M^{(n)} \otimes e_{\mathcal{A}}^{(n)}) \circ \rho_M, \\ &\rho_{M \otimes N} = (id_M \otimes \mathrm{m}_{\mathcal{A}}) \circ (id_M \otimes \tau \otimes id_N) \\ &\circ (\rho_M \otimes \rho_N), \end{aligned} \tag{30}$$

where $\tau : M \otimes N \to N \otimes M$ is the twist, $\mathrm{m}_{\mathcal{A}} : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$ is the multiplication in $\mathcal{A}$.

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